24. Find log 96, log 6, and log 16. Show that log 16 = log 96 − log 6. What property of logarithms does this equality illustrate? What property of exponentiation does this property come from?

25. Find log 32 and log 2. Show that log 32 = 5 log 2. What property of logarithms does this equality illustrate? What property of exponentiation does this property come from?

26. Find log 64 and log 4. Show that log 64 = 3 log 4. What property of logarithms does this equality illustrate? What property of exponentiation does this property come from?

For Problems 27–34, demonstrate numerically the properties of logarithms. Then explain how each result agrees with the definition of logarithm.

27. $\log(0.3 \cdot 0.7) = \log 0.3 + \log 0.7$

28. $\log(7 \cdot 8) = \log 7 + \log 8$

29. $\log(30 \div 5) = \log 30 - \log 5$

30. $\log \frac{2}{8} = \log 2 - \log 8$

31. $\log 2^5 = 5 \log 2$

32. $\log 5^3 = 3 \log 5$

33. $\log \frac{1}{7} = -\log 7$

34. $\log \frac{1}{1000} = -\log 1000$

For Problems 35–44, find the missing number.

35. $\log 7 + \log 3 = \log$ __?__

36. $\log 5 + \log 8 = \log$ __?__

37. $\log 48 - \log 12 = \log$ __?__

38. $\log 4 - \log 20 = \log$ __?__

39. $\log 8 - \log 5 + \log 35 = \log$ __?__

40. $\log 2000 - \log 40 - \log 2 = \log$ __?__

41. $7 \log 2 = \log$ __?__

42. $5 \log 3 = \log$ __?__

43. $\log 125 =$ __?__ $\log 5$

44. $\log 64 =$ __?__ $\log 2$

45. *Logarithm of a Power Property Proof Problem:* Prove algebraically that $\log x^n = n \log x$.

46. *Logarithm of a Quotient Property Proof Problem:* Prove algebraically that $\log \frac{x}{y} = \log x - \log y$.

47. *The Name "Logarithm" Problem:*

a. Before there were calculators, if you had to multiply $27 \cdot 356 \cdot 43 \cdot 592$, you would have to use long multiplication three times to multiply 27 by 356, that answer by 43, and then that answer by 592. Simulate this process on your calculator by multiplying $27 \cdot 356$ and writing the result, then multiplying that answer by 43 and writing the result, and then multiplying that answer by 592 and writing the final result.

b. Before there were calculators, if you had to add 27 + 356 + 43 + 592, you could write the numbers in a column and add *without* writing down any intermediate results. Do this addition column-wise, without using a calculator:

$$\begin{array}{r} 27 \\ 356 \\ 43 \\ +\ 592 \\ \hline \end{array}$$

c. Base-10 logarithms were invented so that strings of numbers could be multiplied by adding their logarithms column-wise. You would look up the logarithms in tables, add these column-wise, and then use the tables backward to find the answer. The computation would look something like this:

$\log 27 \approx 1.43144$

$\log 356 \approx 2.5514$

$\log 43 \approx 1.6335$

$\log 592 \approx 2.7723$

Add these logarithms column-wise, without using a calculator. Simulate finding the product by raising 10 to the exponent you found from adding the logarithms and rounding to four significant digits. Does the result agree with your result in part a?